Question 1

A bicycle manufacturing company must plan its production for three types of bikes: city bike, mountain bike, and electric bike.

The data are as follows:

|  |  |  |  |
| --- | --- | --- | --- |
| **Materials** | **City bikes** | **Mountain bikes** | **Electric bikes** |
| **Aluminium** | 7.5 kg | 8 kg | 8 kg |
| **Carbon Fiber** | 1.5 kg | 1.7 kg | 1.5 kg |
| **Plastic** | 1 kg | 0.8 kg | 1.5 kg |
| **Steel** | 1.5 kg | 2.2 kg | 1.5 kg |
| **Rubber** | 1 kg | 1.3 kg | 1.2 kg |
| **Lithium** | 0 kg | 0 kg | 3.3 kg |

The costs per square meter are as follows:

* Aluminium: 50 €/kg
* Carbon fiber: 100 €/kg
* Plastic: 20 €/kg
* Steel: 30 €/kg
* Rubber: 15 €/kg
* Lithium: 120 €/kg

In addition, each city, mountain and electric bike requires 8 hours, 12 hours and 18 hours of labour respectively. The assembly line is available 24/7 thanks to 5 teams of 20 people. The teams work in 3x8 shifts: the day is divided into three eight-hour slots to which the teams are assigned. Labor costs are €25/hour for two-thirds during the day and €35/hour for one-third at night.

Raw material resources are limited: the company has 600 kg of aluminum, 100 kg of carbon fiber, 50 kg of plastic, 30 kg of steel, 20 kg of rubber and 50 kg of lithium.

City bikes are sold at 400€ each, off-road bikes at 650€ each and electric bikes at 1100€ each.

We want to know the quantities of city, mountain and electric bikes to be manufactured during the month in order to maximize the company's profit within the limits of available resources. The average demand being 80 sales per month, we should not produce more bikes than necessary.

**Q1. Model the problem with a linear program that maximizes the total profit of the company. For each constraint, you will have to write a sentence in French explaining its usefulness.**

Answer:

To model the bike production problem to maximize the total profit of the company, we need to define a linear program (LP) using the data provided.

Decision variables are:

* x1 The number of city bikes produced
* x2 The number of mountain bikes produced
* x3 The number of e-bikes produced

Objective function

We seek to maximize total profit, which is the difference between revenue and cost. Revenue is given by the selling price of the bikes, and costs include the cost of raw materials and the cost of labor.

Revenue:

* City bikes: 400x1
* Mountain bikes: 650x2
* Electric bikes: 1100x3

Raw material costs:

* Aluminium : 7.5x1 + 8x2 + 8x3 ≤ 600
* Carbon Fiber: 1.5x1 + 1.7x2 + 1.5x3 ≤ 100
* Plastic: x1 + 0.8x2 + 1.5x3 ≤ 50
* Steel : 1.5x1 + 2.2x2 + 1.5x3 ≤ 30
* Rubber: x1 + 1.3x2 + 1.2x3 ≤ 20
* Lithium : 3.3x3 ≤ 50

Labor Costs:

* 5 teams of 20 people work 24 hours a day, which gives 5×20×24×30=72000 hours available per month.
* 16 hours per day are paid at €25/hour and 8 hours at €35/hour

16 hours/day×25 €/hour×30 days=12000 €/month

8 hours/day×35 €/hour×30 days=8400 €/month

Labor Cost per Bike:

* City bike: 8 hours x 25€/hour
* Mountain bike: 12 hours x 25€/hour
* Electric bike: 18 hours x 25€/hour

Constraints

1. Availability of raw materials:

* Aluminum : 7.5x1 + 8x2 + 8x3 ≤ 600
* Carbon Fiber: 1.5x1 + 1.7x2 + 1.5x3 ≤ 100
* Plastique : x1 + 0.8x2 + 1.5x3 ≤ 50
* Acier : 1.5x1 + 2.2x2 + 1.5x3 ≤ 30
* Caoutchouc : x1 + 1.3x2 + 1.2x3 ≤ 20
* Lithium : 3.3x3 ≤ 50

1. Availability of Labour:

8x1 + 12x2 + 18x3 ≤ 72000

1. Maximum Demand:

x1 + x2 + x3 ≤ 80

1. Non-negativity:

x1,x2,x3 ≥ 0

Linear program modeling

The problem can be formulated as follows:

Interpretation of constraints:

1. **Aluminum** : Limits the use of aluminium to 600 kg.

2. **Carbon Fiber** : Limits the use of carbon fiber to 100kg.

3. **Plastic** : Limits the use of plastic to 50 kg.

4. **Steel** : Limits the use of steel to 30kg.

5. **Rubber** : Limits the use of rubber to 20kg.

6. **Lithium** : Limits the use of lithium to 50kg.

7. **Labor** : Limits the number of hours of work to 72000 hours per month.

8. **Demand** : Limits the total output to 80 bikes to meet the demand.

9. **Non-negativity** : Ensures that the number of bikes produced is not negative.

Question 2.

Find the optimal solution with one of the following methods: graphical method and/or Excel solver. (Provide the Excel file and/or graph) and answer the following questions:

* How many bikes of each type should be manufactured during the month?
* How many hours of labour would it be necessary to mobilise?

